Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Wednesday 10 October 2018

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA01/01**

Core Mathematics C12

Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. (i) Given that $125\sqrt{5} = 5^a$, find the value of a.

(2)

(ii) Show that $\dfrac{16}{4-\sqrt{8}} = 8 + 4\sqrt{2}$

You must show all stages of your working.

(3)

i) $5^3 = 125$.

$\therefore 5^3 \times 5^{1/2} = 5^{7/2}$

$\therefore a = 7/2$

ii) $\dfrac{16(4+\sqrt{8})}{(4-\sqrt{8})(4+\sqrt{8})}$

$\dfrac{64+16\sqrt{8}}{16-8} = \dfrac{64+16\sqrt{8}}{8}$.

$= 8 + 2\sqrt{8}$

$\sqrt{8} = 2\sqrt{2}$

$\therefore\ 8 + 4\sqrt{2}$ as req.

Leave blank

2. Use algebra to solve the simultaneous equations

$$x + y = 5$$

$$x^2 + x + y^2 = 51$$

You must show all stages of your working.

(7)

(2) $y = 5 - x$

$x^2 + x + (5-x)^2 = 51$

$= x^2 + x + 25 - 10x + x^2 = 51$

$= 2x^2 \cancel{4} -9x - 26 = 0$

$\dfrac{+9 \pm \sqrt{9^2 - 4(2x-26)}}{2\times 2}$

$x = -2$ or $\dfrac{13}{2}$

$y = 5 - x$

$y = 7$ or $\dfrac{-3}{2}$

$\therefore$ when $x = -2$
$y = 7$

when $x = \dfrac{13}{2}$

$y = -\dfrac{3}{2}$

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3. Given that $y = 2x^3 - \frac{5}{3x^2} + 7$, $x \neq 0$, find in its simplest form

(a) $\frac{dy}{dx}$, **(3)**

(b) $\int y \, dx$. **(4)**

(a) $\frac{dy}{dx} = 6x^2 + \frac{10}{3x^3}$

(b) $\int y dx$

$= \int 2x^3 - \frac{5}{3x^2} + 7$

$= \frac{2x^4}{4} - \frac{5}{3x-1}x^{-1} + 7x + c$

$= \frac{1}{2}x^4 + \frac{5}{3}x^{-1} + 7x + c.$

Leave blank

4. A sequence of numbers $u_1, u_2, u_3, \ldots$ satisfies

$$u_n = kn - 3^n$$

where k is a constant.

Given that $u_2 = u_4$

(a) find the value of k **(3)**

(b) evaluate $\sum_{r=1}^{4} u_r$ **(3)**

(a) $u_1 = k - 3^1$

$= k - 3.$

$u_2 = 2k - 3^2$

$= 2k - 9$

$u_3 = \cancel{u_3}\ 3k - 3^3$

$= 3k - 27.$

$u_4 = 4k - 3^4$

$= 4k - 81$

$4k - 81 = 2k - 9.$

$2k = 72$

$k = 36$

(b) $(36 - 3) + (2(36) - 9)$

$+ (3(36) - 27) +$

$(4(36) - 81)$

$= 240$

Leave blank

5. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1 - \frac{1}{2}x\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Hence find the coefficient of x^3 in the expansion of

$$\left(3 + 5x - 2x^2\right)\left(1 - \frac{1}{2}x\right)^{10}$$

(2)

(a) $\binom{10}{0}(1)^{10}\left(\frac{-1}{2}x\right)^0 + \binom{10}{1}\left(\frac{-1}{2}x\right)^1 + \binom{10}{2}\left(\frac{-1}{2}x\right)^2 + \binom{10}{3}\left(\frac{-1}{2}x\right)^3$

$+\binom{10}{4}\left(\frac{-1}{2}x\right)^4$ (not required)

$= 1 - 5x + \frac{45}{4}x^2 - 15x^3, \ldots$

(b) $\left(3+5x-2x^2\right)\left[1-5x+\frac{45}{4}x^2-15x^3\right]$

Co-eff of x^3

$-2x^2 \times -5x = 10x^3$

$\cancel{-2x}\ 5x \times \frac{45}{4}x^2 = \frac{225}{4}x^3$

$3 \times -15x^3 = -45x^3$

$10 + \frac{225}{4} - 45 = \frac{85}{4}$

Leave blank

$y = 2^{-x}$

6. (a) Sketch the graph of $y = \left(\frac{1}{2}\right)^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph crosses the y-axis.

(2)

The table below gives corresponding values of x and y, for $y = \left(\frac{1}{2}\right)^x$

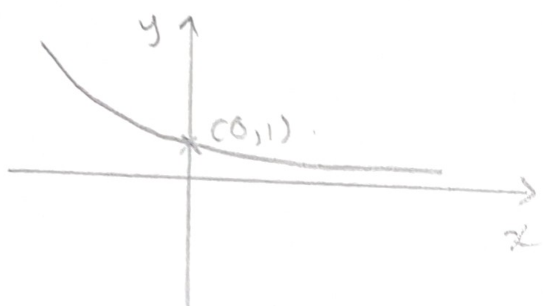
The values of y are rounded to 3 decimal places.

x	−0.9	−0.8	−0.7	−0.6	−0.5
y	1.866	1.741	1.625	1.516	1.414

(b) Use the trapezium rule with all the values of y from the table to find an approximate value for

$$\int_{-0.9}^{-0.5} \left(\frac{1}{2}\right)^x \, dx$$

(3)



(b) $h \Rightarrow$

$-0.8 - -0.9 = \underline{0.1}$

$\frac{1}{2} \times 0.1 \times [1.866 + 1.414 + 2(1.741 + 1.625 + 1.516)]$

$= \underline{\underline{0.652}}$ (3sf)

Leave blank

7. The point A has coordinates $(-1, 5)$ and the point B has coordinates $(4, 1)$.

The line l passes through the points A and B.

(a) Find the gradient of l.
(2)

(b) Find an equation for l, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.
(2)

The point M is the midpoint of AB.

The point C has coordinates $(5, k)$ where k is a constant.

Given that the distance from M to C is $\sqrt{13}$

(c) find the exact possible values of the constant k.
(4)

(a) $(-1,5)$ $(4,1)$

$\frac{y_2-y_1}{x_2-x_1} = \frac{1-5}{4+1} = \frac{-4}{5}$

(b) $y = mx+c$.

$y = -\frac{4}{5}x + c$ at $(-1,5)$

$5 = \frac{4}{5} + c$.

$c = \frac{21}{5}$

$y = -\frac{4}{5}x + \frac{21}{5}$.

$5y + 4x - 21 = 0$.

(c) Finding M

$\frac{-1+4}{2} = \frac{3}{2}$ $\frac{5+1}{2} = 3$.

$\therefore M \Rightarrow (1.5, 3)$.

$(5,k)$ $(1.5,3)$

$\sqrt{(5-1.5)^2+(k-3)^2} = \sqrt{13}$.

$\frac{49}{4} + (k-3)^2 = 13$.

$(k-3)^2 = \frac{3}{4}$

$k-3 = \pm\frac{\sqrt{3}}{2}$

$k = 3 \pm \frac{\sqrt{3}}{2}$

Leave blank

8.

$$f(x) = 2x^3 - 3x^2 + px + q$$

where p and q are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is -6

(a) Use the remainder theorem to show that $p + q = -5$

(2)

Given also that $(x + 2)$ is a factor of $f(x)$,

(b) find the value of p and the value of q.

(3)

(c) Factorise $f(x)$ completely.

(4)

8. $f(1) = -6$.

$2(1)^3 - 3(1)^2 + p(1) + q = -6$.

$2 - 3 + p + q = -6$

$p + q = -5$ as req.

(b) $f(-2) = 0$.

$2(-2)^3 - 3(-2)^2 + p(-2) + q = 0$

$-16 - 12 - 2p + q = 0$.

$-2p + q = 28$.

$-\ p + q = -5$

$-3p = 33$

$\underline{\underline{p = -11}}$

$q = -5 + 11$

$\underline{\underline{q = 6}}$.

(c) $(x+2)(ax^2 + bx + c)$

$\equiv 2x^3 - 3x^2 - 11x + 6$.

$2c = 6$

$\underline{c = 3}$

$ax^3 = 2x^3$ $\quad \underline{a = 2}$

$2ax^2 + bx^2 = -3$.

$4 + b = -3$

$b = -7$.

$\therefore$ another factor $= 2x^2 - 7x + 3$.

Factorising $2x^2 - 7x + 3$

$\Rightarrow (2x - 1)(x - 3)$.

$\therefore f(x) = (x+2)(2x-1)(x-3)$.

Leave blank

9. A car manufacturer currently makes 1000 cars each week.

The manufacturer plans to increase the number of cars it makes each week.

The number of cars made will be increased by 20 each week from 1000 in week 1, to 1020 in week 2, to 1040 in week 3 and so on, until 1500 cars are made in week N.

(a) Find the value of N.

(2)

The car manufacturer then plans to continue to make 1500 cars each week.

(b) Find the total number of cars that will be made in the first 50 weeks starting from and including week 1.

(5)

$a = 1000$
$d = 20$

$a + d(n-1) =$

$1000 + 20(N-1) = 1500$

$N = 26$

(b) Week 1 → 26.

$\frac{26}{2}\left[2(1000) + 20(26-1)\right]$

$= 32500$

From week 27 → 50.

$n = 24$

$24 \times 1500 = 36000$

$36000 + 32500$

$= 68500$

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10.

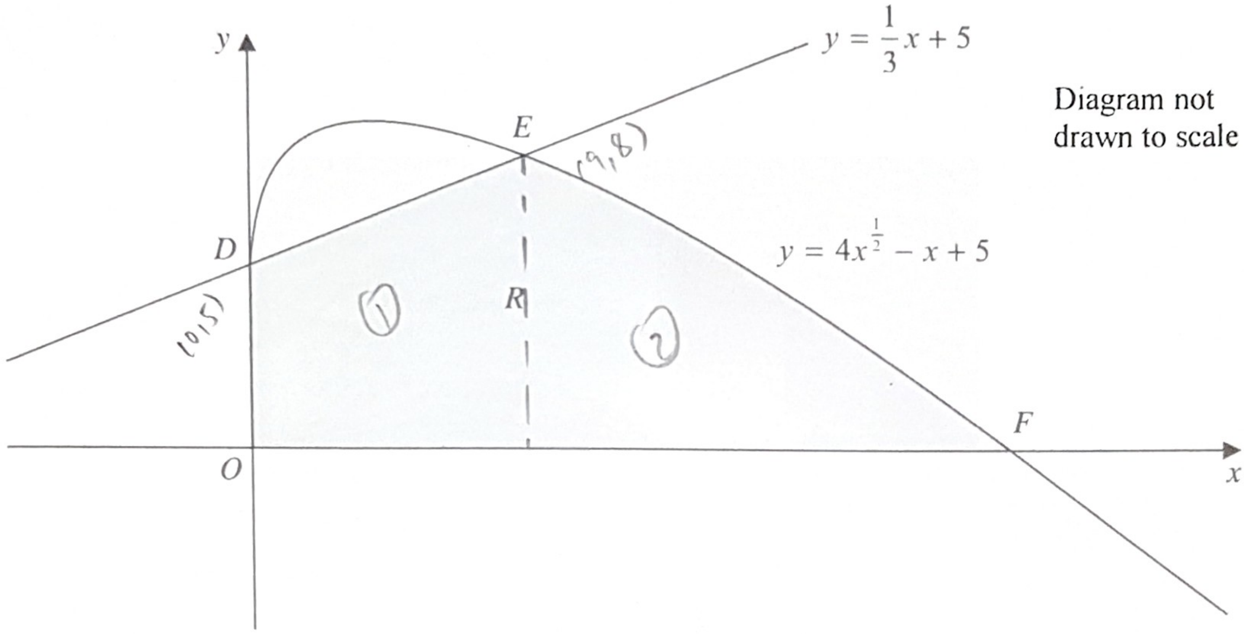


Figure 1

The finite region R, which is shown shaded in Figure 1, is bounded by the coordinate axes, the straight line l with equation $y = \frac{1}{3}x + 5$ and the curve C with equation $y = 4x^{\frac{1}{2}} - x + 5$, $x \geqslant 0$

The line l meets the curve C at the point D on the y-axis and at the point E, as shown in Figure 1.

(a) Use algebra to find the coordinates of the points D and E.

(4)

The curve C crosses the x-axis at the point F.

(b) Verify that the x coordinate of F is 25

(1)

(c) Use algebraic integration to find the exact area of the shaded region R.

(6)

$4x^{1/2} - x + \cancel{5} = \frac{1}{3}x \cancel{+5}$.

$4x^{1/2} = \frac{4}{3}x$.

$x^{1/2} = \frac{1}{3}x$.

$x = \frac{1}{9}x^2$.

$x^2 - 9x = 0$

$x(x-9) = 0$ $x = 0$ $x = 9$.

$y = 5$ when $x = 0$

$y = 8$ when $x = 9$.

Leave blank

Question 10 continued

(b) x int $y=0$.

$4x^{1/2} - x + 5 = 0$

when $x = 25$

$4(25)^{1/2} - (25) + 5$

$= 0$

$\therefore$ x co-ordinate of $F = 25$

(c) ①

$\frac{1}{2} \times 9 \times (8+5)$

$= \frac{117}{2}$

②

$\int_9^{25} 4x^{1/2} - x + 5 \, dx$.

$\left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{2} + 5x \right]_9^{25}$

$\frac{875}{6} - \frac{153}{2}$

$= \frac{208}{3}$

$\frac{208}{3} + \frac{117}{2}$

$= \frac{767}{6}$.

Turn over

11. The equation $7x^2 + 2kx + k^2 = k + 7$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies the inequality

$$6k^2 - 7k - 49 < 0$$

(4)

(b) Find the range of possible values for k.

(4)

$7x^2 + 2kx + k^2 - k - 7 = 0.$

$b^2 - 4ac > 0.$

$(2k)^2 - 4(7)(k^2 - k - 7) > 0$

$4k^2 - 28(k^2 - k - 7) > 0$

$4k^2 - 28k^2 + 28k + 196 > 0.$

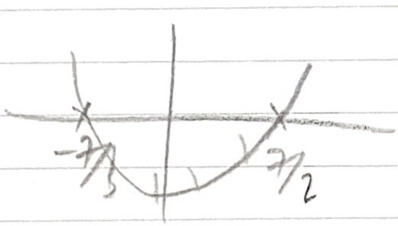
$\frac{-24k^2}{-4} + \frac{28k}{-4} + \frac{196}{-4} > 0.$

$6k^2 - 7k - 49 < 0$ as req.

(b) solve
sketch
Range.

$\frac{7 \pm \sqrt{7^2 - 4(6x - 49)}}{2 \times 6.}$

$k = -\frac{7}{3}$ $\quad k = \frac{7}{2}$



$\therefore \quad -\frac{7}{3} < k < \frac{7}{2}$

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12. (a) Show that the equation

$$6\cos x - 5\tan x = 0$$

may be expressed in the form

$$6\sin^2 x + 5\sin x - 6 = 0$$

(3)

(b) Hence solve for $0 \leqslant \theta < 360°$

$$6\cos(2\theta - 10°) - 5\tan(2\theta - 10°) = 0$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

$6\cos x - \frac{5\sin x}{\cos x} = 0$

$6\cos^2 x - 5\sin x = 0$

$6(1-\sin^2 x) - 5\sin x = 0$

$6 - 6\sin^2 x - 5\sin x = 0$

$6\sin^2 x + 5\sin x - 6 = 0$

as req

(b) let $x = 2\theta - 10°$

$\frac{-5 \pm \sqrt{5^2 - 4(6)(-6)}}{2 \times 6}$

$\sin x = -3/2$ $\sin x = 2/3$

not in range

$\therefore \sin(2\theta - 10) = 2/3$

$2\theta - 10 = 41.8, 138.2, 401.8, 498.2$

$2\theta = 51.8, 148.2, 411.8, 508.2$

$\theta = \underline{25.9}, \underline{74.1}, 205.9, \underline{254.1}$

Leave blank

13. (i) Find the value of x for which

$$4^{3x+2} = 3^{600}$$

giving your answer to 4 significant figures.

(3)

(ii) Given that

$$\log_a (3b - 2) - 2\log_a 5 = 4, \qquad a > 0, a \neq 1, b > \frac{2}{3}$$

find an expression for b in terms of a.

(4)

i) $(3x+2)\log 4 = 600\log 3$

$3x+2 = \dfrac{600\log 3}{\log 4}$

$3x+2 = 475.5$

$3x = 473.5$

$x = \underline{\underline{157.8}}$

(ii) $\log_a\left(\dfrac{3b-2}{5^2}\right) = 4$

$a^4 = \dfrac{3b-2}{25}$

$\dfrac{25a^4+2}{3} = b$

Leave blank

14. The circle C has equation

$$x^2 + y^2 + 16y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that the radius of C is 10

(b) find the value of k.

(2)

The point $A(a, -16)$, where $a > 0$, lies on the circle C. The tangent to C at the point A crosses the x-axis at the point D and crosses the y-axis at the point E.

(c) Find the exact area of triangle ODE.

(7)

$(x+0)^2 + (y+8)^2 - 8^2 + k = 0$

$(x+0)^2 + (y+8)^2 = 64 - k$.

centre $= (0, -8)$

(b) Radius $\Rightarrow$ 10.

$\sqrt{64-k} = 10$.

$64 - k = 100$

$\underline{\underline{k = -36.}}$

(c) $a^2 + (-16)^2 + 16(-16) - 36 = 0$

$a^2 = 36$.

$\underline{a = 6}$

Grad of line $(6, -16)$ $(0, -8)$

$\frac{-8+16}{0-6} = \frac{8}{-6} = \frac{-4}{3}$

$\therefore$ Grad of $\perp$ line $= \frac{3}{4}$.

$y - y_0 = m(x - x_0)$.

$y + 16 = \frac{3}{4}(x - 6)$

<u>X int</u>

$x = 8^2/_3$ $(8^2/_3, 0)$

<u>Y int</u>

$(0, -4^1/_2)$

$\frac{1}{2} \times \frac{41}{2} \times \frac{82}{3} = \frac{1681}{6}$.

Leave blank

15.

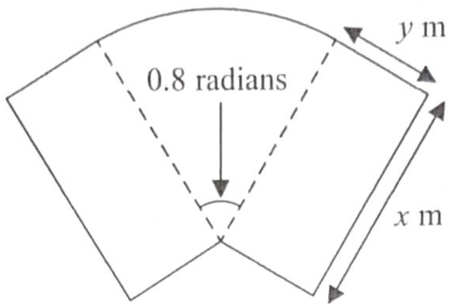

Figure 2

Figure 2 shows a plan for a garden.
The garden consists of two identical rectangles of width y m and length x m, joined to a sector of a circle with radius x m and angle 0.8 radians, as shown in Figure 2.

The area of the garden is 60 m^2.

(a) Show that the perimeter, P m, of the garden is given by

$$P = 2x + \frac{120}{x}$$

(5)

(b) Use calculus to find the exact minimum value for P, giving your answer in the form $a\sqrt{b}$, where a and b are integers.

(4)

(c) Justify that the value of P found in part (b) is the minimum.

(2)

(a) Area

$yx + yx + \left[\frac{1}{2} \times x^2 \times 0.8\right]$

$2yx + 0.4x^2 = 60$

$2yx = 60 - 0.4x^2$

$y = \frac{1}{2x}(60 - 0.4x^2)$

$y = \frac{30}{x} - 0.2x$

Perimeter

$2(x + y + y) + x(0.8)$

$2x + 4\left(\frac{30}{x} - 0.2x\right) + 0.8x$

$2x + \frac{120}{x} - \cancel{0.8x} + \cancel{0.8x}$

$P = 2x + \frac{120}{x}$ as req

Leave blank

Question 15 continued

(b) $\frac{dP}{dx} = 2 - \frac{120}{x^2}$

$\frac{dP}{dx} = 0.$

$2 = \frac{120}{x^2}$

$x^2 = 60$

$x = \pm 2\sqrt{15}.$

Since x is a length

$x > 0 \therefore x = \underline{\underline{2\sqrt{15}}}$

(c) $\frac{d^2P}{dx^2} = \frac{240}{x^3}$

when $x = 2\sqrt{15}$

$\frac{d^2P}{dx^2} > 0 \therefore$ min value of x found.

Leave blank

16. The first three terms of a geometric series are $(k + 5)$, k and $(2k - 24)$ respectively, where k is a constant.

(a) Show that $k^2 - 14k - 120 = 0$

(3)

(b) Hence find the possible values of k.

(2)

(c) Given that the series is convergent, find

(i) the common ratio,

(ii) the sum to infinity.

(4)

$a = k+5$.

$ar = k$.

$ar^2 = (2k-24)$

$$\frac{(2k-24)}{k} = \frac{k}{k+5}$$

$(k+5)(2k-24) = k^2$.

$2(k+5)(k-12) = k^2$.

$2[k^2 - 12k + 5k - 60] = k^2$

$2[k^2 - 7k - 60] = k^2$

$2k^2 - 14k - 120 - k^2 = 0$

$k^2 - 14k - 120 = 0$ as req.

(b) $$\frac{14 \pm \sqrt{14^2 - 4(-120)}}{2 \times 1}$$

$k = -6$ or 20.

(c) Since series is convergent

$k = 20$.

$20 + 5 = a = \underline{\underline{25}}$

$$r = \frac{20}{25} = \frac{4}{5}.$$

$$S_\infty = \frac{a}{1-r} = \frac{25}{1-0.6} =$$

$= \underline{\underline{125}}$